UNIT – II

Conductors and Dielectrics

2.1) Poisson's and Laplace's Equations:

From the Gauss law we know that

$\int D.ds = Q$ (1)

A body containing a charge density ρ uniformly distributed over the body. Then charge of that body is given by

$Q = \int \rho \, dv$ (2)

$\int D.ds = \int \rho \, dv$ (3)

This is integral form of Gauss law.

As per the divergence theorem

$\int D.ds = \int \nabla.\mathbf{D} \, dv$ (4)

$\nabla.\mathbf{D} = \rho$ (5)

This is known as point form or vector form or polar form. This is also known as Maxwell's first equation.

$D = \varepsilon E$ (6)

$\nabla. \varepsilon E = \rho$

$\nabla. E = \rho/\varepsilon$ (7)

We know that E is negative potential medium

$E = -\nabla V$ (8)

From equations 7 and 8

$\nabla. (-\nabla V) = \rho/\varepsilon$

$\nabla^2 V = -\rho/\varepsilon$ (9)

Which is known as Poisson's equation in static electric field.

Consider a charge free region (insulator) the value of $\rho = 0$, since there is no free charges in dielectrics or insulators.

$\nabla^2 V = 0$

This is known as Laplace's equation.

Cartesian coordinate system

$\nabla^2 V = \frac{\partial 2}{\partial x} V$

Cylindrical coordinate system

Spherical coordinate system

2.2) Electric Dipole:

It is defined as two equal and opposite charges separated by a small distance.

2.3) Electric Dipole moment:

It is defined as product of charge Q and distance between the two charges. It is a vector since length is in vector, length vector is directed from negative to positive charge, therefore dipole moment is from negative to positive charge.

m = Ql

$l = l\ u_r$

u_r is unit vector directed from negative to positive charge. The unit of dipole moment is Cm

2.4) Potential due to electric dipole:

we have to find the potential at P which is at a distance r from the centre of the dipole. The distance from +Q and –Q to the point P are r_1 and r_2

$r_1 = r - \frac{l}{2}\cos\theta$

$r_2 = r + \frac{l}{2}\cos\theta$

Let V_1 be the potential due to +q, V_2 be the potential due to –Q.

$V_1 = \frac{Q}{4\pi\varepsilon r1}$

$V_2 = \frac{-Q}{4\pi\varepsilon r2}$

By the superposition theorem, total potential due to dipole

$V = V_1 + V_2$

$V = \frac{Q}{4\pi\epsilon}\frac{r2 - r1}{r1r2}$

$r2 - r1 = l\cos\theta$

$r2r1 = r^2$

$V = \frac{Ql\cos\theta}{4\pi r2}$

M is magnetic dipole moment

We know that

$E = -\nabla V$

2.5) Torque experienced by Dipole in uniform Electric Field:

There are two charges +Q and –Q forming a dipole, placed in uniform E. Each charge will experience a force equal in magnitude QE but oppositely directed and resultant force experienced by dipole zero because as F1 and F2 neutralize each other but these forces form a couple whose torque is equal to magnitude oppose into perpendicular distance between the couple charges.

T = dxF

$\sin\theta = d/l$

$d = l \sin\theta$

$T = l \sin\theta \, . \, F$

$T = Ql \sin\theta \, . \, E$

T=mxE

The torque is maximum when E and dipole moment are perpendicular to each other. The torque is minimum when E and dipole moment are parallel. So we conclude that dipole in uniform E does not experience translational forces. It experiences a force tending aligned the dipole axes with the E.

2.6) Capacitor:

It is an electric device consisting of 2 conductors separated by insulator or dielectric medium.

2.6.1) Capacitance:

It is the property of capacitor which stores electrical energy in electric field. It is the property of capacitor which oppose the sudden change in voltage. It is the ratio of charge on one of its conductors to the potential difference between them.

$C = \frac{Q}{V}$

2.6.2) Capacitance of Parallel plate capacitor:

Consider a parallel plate capacitor shown in fig. The space between 2 plates is filled with dielectric. The charge of each plate is Q and area of each plate is A and distance between 2 plates is d. We know that electric flux density

$D = Q/A$ (1)

$E = D/\varepsilon$ (2)

$E = Q/(A\varepsilon)$

$E = V/d$ (3)

Equating equations 2 and 3

$V/d = Q/(A\varepsilon)$

$C = \varepsilon A/d$

Capacitance is directly proportional to area and inversely proportional to distance between plates.

2.6.3) Capacitance of Parallel plate capacitor with Multimedia:

Consider a parallel plate capacitor with 2 media as shown. The thickness of media is d1 and other is d2. The potential is applied between media V1 and V2.

Apply KVL to a parallel plate capacitor

$V = V_1+V_2$

$V = E_1d_1+ E_1d_2$

$V = d_1D/\varepsilon_1 + d_2D/\varepsilon_2$

$V = (Q/A)[d_1/\varepsilon_1 + d_2/\varepsilon_2]$

$C = Q/V = A/[d_1/\varepsilon_1 + d_2/\varepsilon_2]$

$C = A/[d_1/\varepsilon_1 + d_2/\varepsilon_2+ \ldots. + d_n/\varepsilon_n]$

2.6.4) Capacitance of Spherical capacitor:

Consider a spherical capacitor shown. The space between 2 spherical shells is filled with dielectric of permittivity ε_r and a and b radius of inner and outer spherical shells. Construct a Gaussian spherical cell of radius r and apply Gauss law

$\varphi = Qe$

$\int D.ds = Q$

$D\ 4\pi r^2 = Q$

$D = Q / (4\pi r^2)$

$E = Q / (4\pi \varepsilon r^2)$

$V_{ab} = - \int_b^a E\ .dl$

$= \int_a^b \frac{Q}{4\pi\epsilon r^2}$ dr

$= \frac{Q}{4\pi\varepsilon}[\frac{1}{a} - \frac{1}{b}]$

$C = \frac{Q}{V_{ab}} = 4\pi\varepsilon$ (ab)/(a-b)

2.6.5) Capacitance of cylindrical capacitor:

Consider a cylindrical capacitor as shown. Radius of inner and outer cylinders are a and b. The space between 2 cylinders are filled with dielectric of permittivity ε_r. The conductor of charge λ C/m. Construct a Gaussian cylinder and apply Gauss Law.

$\varphi = Qe$

$\int D.ds = \lambda l$

$D\ 2\pi\rho l = \lambda l$

$D = \lambda/(2\pi\rho)$

$E = D/\varepsilon = \lambda/(2\pi\varepsilon\rho)$

$V_{ab} = -\int_b^a E\ .dl$

$= \int_a^b \frac{\lambda}{2\pi\varepsilon\rho}\ d\rho$

$= \frac{\lambda}{2\pi\varepsilon} \ln\frac{b}{a}$

$Q = \lambda l$

$C = \frac{Q}{V_{ab}} = \frac{2\pi\varepsilon}{\ln(\frac{b}{a})}$

2.7) Energy stored in a capacitor:

A capacitor is charged from a dc source as shown. The voltage across the capacitor is V in the closed loop. The V across the capacitor opposes the supply voltage, the workdone by the source is against the opposition of V is converted into energy. We know that potential is the workdone by the charge. Let dw is the workdone to establish a charge dq by the deficition of potential

$V = \frac{dw}{dq}$

$dw = Vdq$

$= \frac{q}{c}\ dq$

Then the total workdone to establish a charge +Q can be obtained by integration

$W = \int_0^Q \frac{q}{c}\, dq$

$= \frac{1}{2}\frac{Q^2}{c}$

$= \frac{1}{2} CV^2$

$= \frac{1}{2} QV$

2.8) Energy Density:

The energy stored in a capacitor is

$W = \frac{1}{2} CV^2$

We know that $C = \frac{\epsilon A}{d}$

$W = \frac{1}{2} V^2 \frac{\epsilon A}{d} = \frac{1}{2} \epsilon (\frac{V}{d})^2 \text{ Ad}$

$W = \frac{1}{2} \epsilon E^2$ Volume

$\frac{Energy\ stored}{volume} = \frac{1}{2} \epsilon E^2$

Energy density $= \frac{1}{2} \epsilon E^2 = \frac{1}{2}$ DE

Energy stored $= \int_v \frac{1}{2} DE\ dv$

Energy density due to n charges:

$W = \frac{1}{2} QV$

$W = \frac{1}{2} \{Q_1V_1 + Q_2V_{2+.........+} Q_nV_n\}$

2.9) Current and current density:

Current through a given area is the electric charge passing through the area per unit time.

$I = - \frac{dQ}{dt}$

'-' indicates the opposite direction of electrons to the current.

Conduction current or Drift current:

The current flow due to the flow of free electrons in the conductor under the influence of applied voltage is called drift current. It obeys Ohms law.

Convection Current or displacement current:

The current due to the flow of charge under the influence of electric field is called convection current. It does not obey Ohms law.

Diffusion Current:

The current due to the movement of free electrons and free holes in semi conductor is called diffusion current.

Current Density:

The amount of current passing through a conductor is normal to the area of cross section / unit area is called current density. It is given by J.

When a steady current is passing through conductor, the current density is uniform and conduction has uniform cross section but J is different at different points if the conduction is non uniform cross section. The current density is represented by vector J. The unit is A/m^2

$J = \lim_{S \to \Delta s} \frac{\Delta J}{\Delta S}$

$= I/S$

$= dI/dS$

$dI = J\ dS$

$I = \int J.dS$

Depending upon the current is produced there are 2 types of J

1. Conduction current density or point form of Ohms law
2. Convection current density

Conduction current density or Point form of Ohms law:

Conduction current requires conductors.

Ohms law:

The current flowing through a linear circuit is directly proportional to impressed voltage provided the temperature is kept constant.

$I \alpha V$

$I = GV$

$I = V/R$

We know $R = \rho l/A$

$I = \frac{VA}{\rho l}$

$\frac{I}{A} = \frac{1}{\rho}\frac{V}{l}$

J = σ. E

Which is known as point form of Ohms law for insulators or dielectrics.

J = 0 conduction current cannot flow through free space.

Convection current density:

Convection current does not involve conductor and it does not obey Ohms law. It occurs and current flowing through a insulating medium or through liquid or through vaccum. Consider a current filament as shown. There is a flow of charge density ρ_v at velocity v along y-axis.

$v = v_y \, a_y$

The current through the filament

$\Delta I = \frac{\Delta Q}{\Delta t}$

We know $\rho_v = \frac{\Delta Q}{\Delta v}$

$\Delta Q = \rho_v \Delta V$

$\Delta I = \rho_v \frac{\Delta V}{\Delta t} = \rho_v \frac{\Delta S \, \Delta l}{\Delta t}$

$\frac{\Delta I}{\Delta S} = \rho_v \frac{\Delta l}{\Delta t}$

$J = \rho_v \, v_y \, a_y$

Here ΔI is the convection current and J is the convection current density.

2.10) Continuity Equation:

Continuity equation of charge works on the principle of law of conservation of charge. It states that the charge can neither be created nor destroyed. We know that current is rate of flow of charge.

$I = -\frac{dQ}{dt}$ (1)

'-' indicates the opposite direction of electrons to the current.

We know that volume charge density

$\rho_v = \frac{dQ}{dV}$

$Q = \int \rho_v \, dV$ (2)

From equations 1 & 2

I = - $\int \frac{d\rho_v}{dt}\, dV$ (3)

We know

I = $\int J.dS$ (4)

From 3 & 4

$\int J.dS$ = - $\int \frac{d\rho_v}{dt}\, dV$

$\int \nabla.J\, dV$ = - $\int \frac{d\rho_v}{dt}\, dV$

$\nabla.J$ = - $\frac{d\rho_v}{dt}$

2.11) Boundary conditions between conductors and free space(Dielectric):

First Boundary condition:

Bigger source a boundary formed by conductor and free space, the charge cannot reside inside a conductor since they repel each other and finally they reach the boundary of the conductor. Construct a rectangular path ABCDA as shown. We know that electric field is conservative field

$\int E.dl = 0$

$\int_{AB} E.dl + \int_{BC} E.dl + \int_{CD} E.dl + \int_{DA} E.dl = 0$

$E_{t1}\, \Delta l - E_{h1} \frac{\Delta h}{2} + 0 + 0 + 0 + E_{n1} \frac{\Delta h}{2} = 0$

$E_{t1} = 0$

$E = E_{t1} + E_{n1}$

$= E_{n1}$

Electric field is always normal to the surface of the conductor.

Second boundary condition:

Construct a pill box and apply Gauss law to the pill box.

$\varphi = Q$

$\int D.ds = Q$

$\int_{top} D.ds + \int_{bottem} D.ds + \int_{lateral} D.ds = \sigma\, dS$

$D_{n1} = \sigma$

Normal component of flux density is equal to the normal flux density.

Properties of Conductors:

Electric field inside the conductor is zero.

Electric field is always normal to the surface of the conductor.

The value of electric flux density is equal to surface charge density.

The tangential component of electric field is zero.

2.12) Conductors & Dielectrics:

Conductor is one in which the outer electrons of an atom is easily detachable and migrate with application of weak Electric field.

A dielectric is one in which the electrons are rigidly bounded to their nucleus, so the ordinary electric field will not be able to detach them away. The dielectric placed in electrostatic field will be subjected to electro static induction. The electric field will twisted and strain the molecules to orient the positive charges in the direction of electric field and negative charges oppositely. If the electric field strength is too high the dielectric will break down cease to beam insulator.

2.12.1) Types of Dielectrics:

1. Polar dielectrics
2. Non-polar dielectrics

1. Polar dielectrics:

In polar dielectrics the molecules from dipoles even in absence of electric field. Even in absence of electric field, the dipoles are disposed at random the resultant electric field is zero. On the application of electric field the dipoles rearranged themselves so that their axes are aligned with the applied field. The electric field will twist and strain the molecules to orient the positive charges in the direction of electric field and negative charges oppositely. This shifting results an instantaneous current called displacement current which causes in very small fraction of seconds.

Eg: water, ether, ammonia

2. Non-Polar dielectrics:

In these dielectrics the positive and negative elements in the uncharged conditions are closed to each other that their action is neutral. In the application of electric field will strict the positive and negative charges lightly with in the molecules to give rise to dipole.

Eg: H, O etc

2.13) Polarization:

The elastic shifting of charged clouds in an atom of dielectric material when it is subjected to an electric field is called polarization. It is defined as movement of dipole.

P = $\frac{m}{V}$

If there are n dipoles the volume then total dipole moment is

m = $m_1+m_2+\ldots+m_n\Delta V$

m = $\sum_{i=1}^{n\Delta V} m_i$

polarization = total displacement / volume

$$= \frac{\sum_{i=1}^{n\Delta V} m_i}{V}$$

2.14) Dielectric Parameters:

Consider a dielectric material cutting the form of a slab of permittivity ϵ as shown and placed in uniform electric field. The effect of field due to polarize the dielectric inducing atomic dipole through out the volume of specimen in the alignment with the electric field. Consequently neutralization of equal and opposite charge inside the dielectric charges reside on the slab and form dipole.

Polarization P = $\frac{Ql}{V} = \frac{Ql}{Al} = \frac{Q}{A} = \sigma_p\, u_l$

σ_p is surface charge density.

The internal field $E_i = E_a + E^1$

Where E_a is applied field, E^1 is field induced in the slab which is opposite to that of applied field.

$E^1 = -\frac{\sigma_p}{\epsilon_0} u_l$

$= -\frac{P}{\epsilon_0} u_l$

$E_i = E_a - \frac{P}{\epsilon_0}$

$E_a = E_i + \frac{P}{\epsilon_0}$

$\epsilon_0 E_a = \epsilon_0 E_i + P$

D = $\epsilon_0 E_i + P$ (1)

$P \alpha E_i$

P = $\epsilon_0 \psi_p E_i$

D = $\epsilon_0 E_i + \epsilon_0 \psi_p E_i$

D = $\epsilon_0 E_i(1+ \psi_p)$

$D = \epsilon_0\ \epsilon_r\ E_i$

Susceptability (ψ_e) :

Number of dipoles induced by unit volume under the influence of unit strength electric field in a material is known as electrical susceptibility.

$\psi_e = \epsilon_r - 1$

Susceptability is one less than relative permittivity. For linear dielectric,

Polarization $\alpha\ E_i$

$P = \epsilon_0\ \psi_e E_i$ (1)

We know that $D = \epsilon_0\ \epsilon_r\ E_i$

$D = \epsilon_0(1+\psi_e)\ E_i$ (2)

From equations 1 & 2

$\frac{P}{D} = \frac{\psi_e}{(1+\psi_e)} = \frac{\psi_e}{\epsilon_r}$ (3)

2.15) Dielectric Boundary conditions:

First boundary condition:

When the flux lines are flow through single medium they are continuous. If they go through boundary formed by two dielectrics they get reflected. First boundary condition deals with electric field intensity.

E_1 and E_2 are electric field in medium 1 and 2 respectively. Construct a rectangular path ABCDA as shown. And apply conservative property for the rectangular loop ABCDA.

$\int E.dl = 0$

$\int_{AB} E.dl + \int_{BC} E.dl + \int_{CD} E.dl + \int_{DA} E.dl = 0$

$E_{t1}\ \Delta l - E_{n1}\frac{\Delta h}{2} - E_{n2}\frac{\Delta h}{2} - E_{t2}\ \Delta l + E_{n2}\frac{\Delta h}{2} + E_{n1}\frac{\Delta h}{2} = 0$

$E_{t1} = E_{t2}$ (1)

At the boundary the tangent along components of electric field vectors are equal.

$\sin\theta_1 = \frac{Et1}{E_1}$

$E_{t1} = E_1 \sin\theta_1$ (2)

$E_{t2} = E_2 \sin\theta_2$ (3)

$E_1 \sin\theta_1 = E_2 \sin\theta_2$ (4)

Second boundary equations:

D_{n1} and D_{n2} are normal components of flux density vectors in medium 1 and 2 respectively. An infinite sheet with charge density σ C/m^2 is at the boundary. Second boundary condition deals with flux density. Construct the pill box at the boundary as shown. Apply Gauss's law

Flux enter the pill box $= D_{n2}\, dS$

Flux leave the pill box $= D_{n1}\, dS$

Net flux in the pill box $= D_{n2}\, dS - D_{n1}\, dS = \sigma\, dS$

$D_{n2} - D_{n1} = \sigma$

If the charge sheet is not present then

$D_{n2} - D_{n1} = 0$

$D_{n2} = D_{n1}$ (5)

This is known as second boundary condition.

$\cos\theta_1 = \frac{D_{n1}}{D_1}$

$D_{n1} = D_1 \cos\theta_1$ (6)

$\cos\theta_2 = \frac{D_{n2}}{D_2}$

$D_{n2} = D_2 \cos\theta_2$ (7)

$D_1 \cos\theta_1 = D_2 \cos\theta_2$ (8)

$\frac{E_1 sin\theta_1}{D_1 cos\theta_1} = \frac{E_2 sin\theta_2}{D_2 cos\theta_2}$

$D_1 = \epsilon_0 \epsilon_{r1} E_1$

$D_2 = \epsilon_0 \epsilon_{r2} E_2$

$\frac{\tan\theta_2}{\tan\theta_1} = \frac{\epsilon_{r2}}{\epsilon_{r1}}$

θ_1 is angle of emergence.

θ_2 is angle of incidence.

This is relation between two dielectric surfaces.

UNIT III

MAGNETOSTATICS

3.1. Steady current (or) D.C current (or) Time invariant current:

The motion of charges is at a constant rate with a time is called steady current. Magneto statics deals with magnetic field produced by steady current.

3.2. Magnetic field:

A static magnetic field can be produced from a permanent magnetic (or) a current carrying conductor. A steady current flowing in a straight conductor produces a magnetic field around it. The field exists as concentric circles having centres at the axis of the conductor.

If we hold the current carrying conductor by the right hand so that the thumb points the direction of current flow,the the fingers point the direction of magnetic field. The unit of magnetic flux is Weber.

1 Wb = 10^8 maxwells

3.3. Magnetic flux density (B):

The magnetic flux per unit area is called magnetic flux density. The unit of magnetic flux density is Tesla (or) Wb/m^2.

The magnitude and direction of magnetic flux density due to current carrying conductor is given by Biot-Savart's law.

$$B = \frac{d\phi}{ds}$$

$$d\phi = B.ds$$

$$\phi = \int_s B.ds$$

3.4. Magnetic field intensity:

The magnetic field intensity at any point is the force experienced by a unit north pole of one weber strength when placed at that point. Unit is N/Wb, A/m (or) AT/m. It is denoted by $\overline{H}$.

3.5. Magneto motive force (mmf):

Mmf is produced when an electric current flows through a coil of several turns. The mmf depends on the current and number of turns. Mmf produces flux in a magnetic circuit. The unit of mmf is Ampere-turns.

3.6. Reluctance (s):

Reluctance is defined as the ratio of mmf to the flux produced. Reluctance is similar to the resistance in a electric circuit.

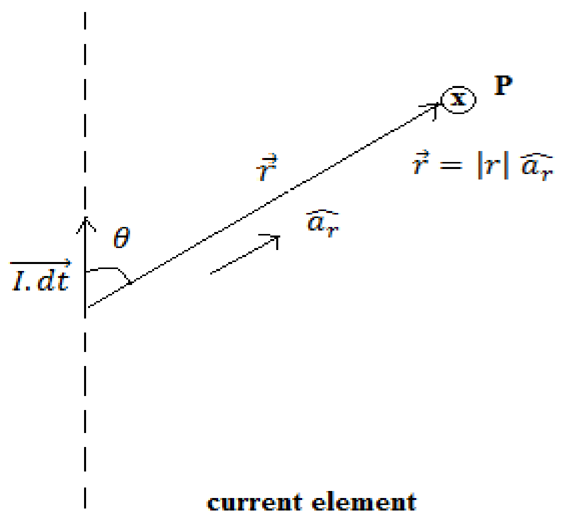
Reluctance is directly proportional to the length of the magnetic path and inversely proportional to the cross-sectional area of the path. The reciprocal of reluctance is called Permeance.

Mmf= Reluctance X flux

Reluctance = $\frac{\text{mmf}}{\text{flux}}$

$S=\frac{l}{\mu_0 \mu_r A}$

3.7. Biot-Savart's law:



Steady current flowing through a straight conductor produces magnetic field in the form of concentric circles.The magnetic field intensity is given by Biot-Savart's law.

A straight conductor is assumed to be formed by several segments. Such segment is called current element. Current element is vector defined as $\overrightarrow{\text{I.dl}}$.

Let the magnetic field intensity at P due to current element $\overrightarrow{\text{I.dl}}$ be $\overrightarrow{\text{dH}}$. The point P is at a distance 'r' m from the current element.

According to Biot-Savort's law, the magnitude of dH is

1) Directly proportional to the current element.
2) Inversely proportional to the square of the distance.
3) Directly proportional to the sine of the angle between current element and distance vector.

$\widehat{a_r}$ is the unit vector normal to the plane of the paper.

1) $|dH| \propto |I\,dl|$
2) $|dH| \propto \frac{1}{|r|^2}$

3) $|dH| \propto \sin\theta$

$|dH| \propto \frac{|I\, dl| \sin\theta}{|r|^2}$

Constant in M.K.S unit is $\frac{1}{4\pi}$

$|dH| = \frac{|Idl| \sin\theta}{4\pi |r|^2}$

$\overrightarrow{dH} = |dH| \widehat{a_r}$

$\overrightarrow{dH} = \frac{|I\, dl| \sin\theta}{4\pi |r|^2} \widehat{a_r}$ since $|r| \widehat{a_r} = \vec{r}$

$\widehat{a_r} = \frac{\vec{r}}{|r|}$

$= \frac{|I\, dl| \sin\theta}{4\pi |r|^2} \frac{\vec{r}}{|r|}$

$= \frac{|I\, dl| \vec{r} \sin\theta}{4\pi |r|^3}$

$\overrightarrow{dH} = \frac{\overrightarrow{Idl} \, X \, \vec{r}}{4\pi |r|^3}$

H due to entire conductor can be obtained by integration.

$\vec{H} = \int \frac{\overrightarrow{I\, dl} \, X \, \vec{r}}{4\pi |r|^3}$

$\vec{H} = \frac{\int \overrightarrow{I\, dl} \, X \, \vec{r}}{4\pi |r|^3}$

$\vec{H} = \frac{\overrightarrow{Il} \, X \, \vec{r}}{4\pi |r|^3}$

$\vec{B} = \mu H$ where μ = permeability

$\mu = \mu_0 \mu_r$

μ_0 = permeability of free space

μ_r = relative permeability

$\vec{B} = \frac{\mu}{4\pi |r|^3} \, S \overrightarrow{I\, dl} \, X \, \vec{r}$

$$\vec{B} = \frac{\mu_0 \mu_r}{4\pi} \frac{(\vec{Il} \, X \, \vec{r})}{|r|^3}$$

For air(or) free space $\mu_r = 1$

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{(\vec{Il} \, X \, \vec{r})}{|r|^3}$$

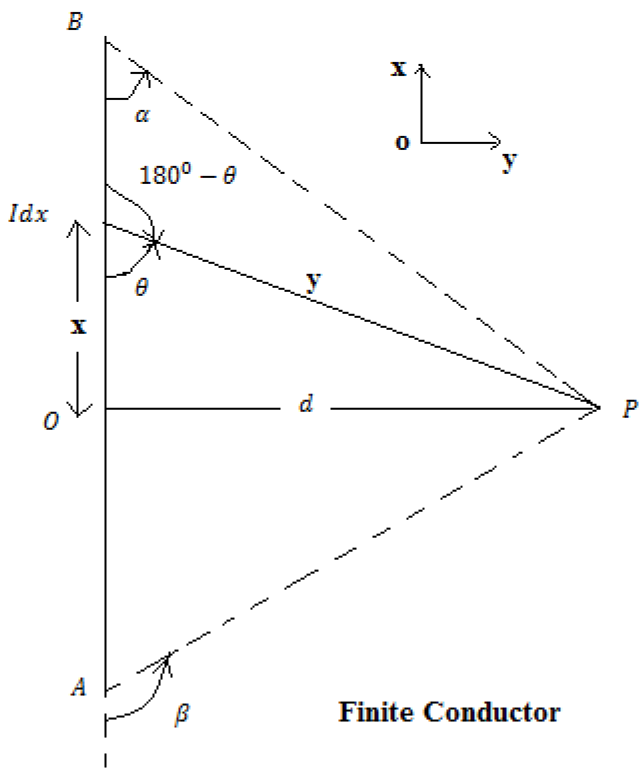
Since $\mu_0 = 4\pi \, X \, 10^{-7}$ Henry/m

$$\vec{B} = 10^{-7} \frac{(\vec{Il} \, X \, \vec{r})}{|r|^3} \quad \text{Wb/m}^2$$

3.8. H due to finite conductor and infinite conductor:

We have to determine H due to a finite current carrying conductor at P. P is at a distance ‘d’ m from the origin.

Consider a current element Idx at a distance ‘x’ m from the origin. The distance vector between current element vector and point P is the vector $\vec{r}$.



Finite Conductor

Let dH be the field intensity due to current element Idx which is at a distance ‘x’ m from the origin.

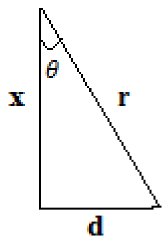
By Biot-Savart’s law,

$$\overrightarrow{dH} = \frac{\overrightarrow{Idl} \, X \, \vec{r}}{4\pi|r|^3}$$

$$|dH| = \frac{Idx|r|\sin(\pi-\theta)}{4\pi|r|^3}$$

$$dH = \frac{Idx \, \sin\theta}{4\pi|r|^2}$$

From Δle , (1)



$$\tan\theta = \frac{d}{x}$$

$$x = d\cot\theta$$

$$dx = -d\,\mathrm{cosec}^2\theta \quad (2)$$

$$\sin\theta = \frac{d}{r}$$

$$r = d\,\mathrm{cosec}\theta \quad (3)$$

From (1),(2) and (3)

$$dH = \frac{-Id\mathrm{cosec}^2\theta \, d\theta \, \sin\theta}{4\pi \, d^2 \, \mathrm{cosec}^2\theta}$$

$$dH = -\frac{I}{4\pi d}\sin\theta \, d\theta$$

Total magnetic field strength is obtained by integration,

$$H = \int_{\beta}^{\alpha} -\frac{I}{4\pi d}\sin\theta \, d\theta$$

$$= -\frac{I}{4\pi d}\int_{\beta}^{\alpha}\sin\theta \, d\theta$$

$$= \frac{I}{4\pi d}(\cos\alpha - \cos\beta)$$

$$\vec{H} = |H|\widehat{a_r}$$

$$\vec{H} = \frac{I}{4\pi d}(\cos\alpha - \cos\beta)\hat{a_r}$$

As length tends to $\infty, \propto \rightarrow 0, \beta \rightarrow 180^0$

$$|H| = \frac{I}{4\pi d}(\cos 0 - \cos 180^0)$$

$$= \frac{I}{4\pi d}(1 - (-1))$$

$$|H| = \frac{I}{2\pi d} \text{ A/m}$$

$$\vec{H} = \frac{I}{2\pi d}\ \hat{a_r} \text{ A/m}$$

From this equation,it can be seen that the magnetic field intensity is inversely proportional to the distance.

$$\vec{B} = \frac{\mu}{2\pi}\frac{I}{d}\ \hat{a_r} \text{ Wb/m}^2$$

Problems:

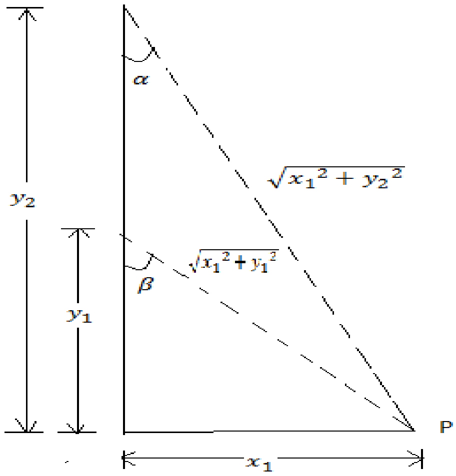
1. A thin linear conductor of length ‘l’ and carrying a current ‘I’ is coincident with y-axis. One end of the conductor is at y, and other end is at y_2 from the origin. Using Biot-Savart’s law,show that the magnetic flux density due to the conductor at a point on the x-axis at a distance x_1 from the origin is $B = \frac{\mu_0 I}{4\pi x_1}\left[\frac{y_2}{\sqrt{{x_1}^2+{y_2}^2}} - \frac{y_1}{\sqrt{{x_1}^2+{y_1}^2}}\right]$.

   Sol: $B = \frac{\mu_0 I}{4\pi x_1}(\cos\alpha - \cos\beta)$

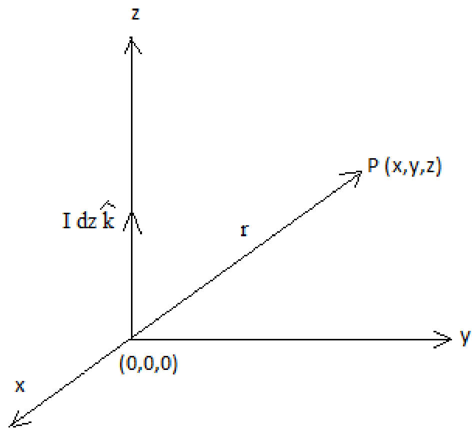
   $$\cos\alpha = \frac{y_2}{\sqrt{{x_1}^2+{y_2}^2}}$$

   $$\cos\beta = \frac{y_1}{\sqrt{{x_1}^2+{y_1}^2}}$$

   Therefore, $B = \frac{\mu_0 I}{4\pi x_1}\left[\frac{y_2}{\sqrt{{x_1}^2+{y_2}^2}} - \frac{y_1}{\sqrt{{x_1}^2+{y_1}^2}}\right]$



2. A current element kept at the origin is coincident with the z-axis. The element is directed towards the positive z-direction. Find magnetic field intensity at a distance 'r' meters from the origin.
   Sol:



$$\vec{r} = x_i + y_j + z_k$$

$$|r| = \sqrt{x^2 + y^2 + z^2} = (x^2 + y^2 + z^2)^{\frac{3}{2}}$$

$$\overrightarrow{dH} = \frac{\overrightarrow{Idz} \, X \, \vec{r}}{4\pi|r|^3}$$

$$\overrightarrow{dH} = \frac{Idz \, k \, X \, (x_i + y_j + z_k)}{4\pi(x^2+y^2+z^2)^{\frac{3}{2}}}$$

$$\overrightarrow{dH} = \frac{Idz \, (x_j - y_i)}{4\pi(x^2+y^2+z^2)^{\frac{3}{2}}}$$

3. A steady current of I amps flows in a conductor bent in the form of a square loop of side 'a'. Find the magnetic field intensity at the center of the loop.

Sol:

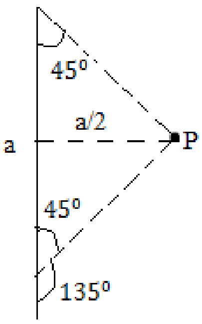


Fig. a

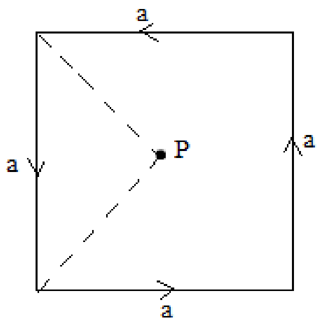


Fig. b

From fig a, the magnetic field intensity due to one conductor is

$$H_1 = \frac{I}{4\pi d}(\cos\alpha - \cos\beta)$$

$$= \frac{I}{4\pi \frac{a}{2}}(\cos 45^0 - \cos 135^0)$$

$$= \frac{I}{2\pi a}\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)$$

$$= \frac{I}{2\pi a}\left(\frac{2}{\sqrt{2}}\right)$$

$$= \frac{I}{\sqrt{2}\pi a}$$

Magnetic field intensity at the center of the loop

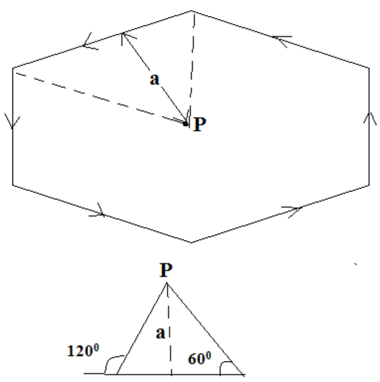
$$H = 4H_1 = \frac{2\sqrt{2}\, I}{\pi a}$$

4. A steady current ‘I’ A flows in a conductor bent in the form of a hexagon. Find the intensity at the center of the loop. The distance between center and each side is ‘a’ m.
Sol: Let H be the magnetic field intensity due to one side.

$$H_1 = \frac{I}{4\pi d}(\cos\alpha - \cos\beta)$$

$$H_1 = \frac{I}{4\pi a}(\cos 60^0 - \cos 120^0)$$

$$= \frac{I}{4\pi a}$$

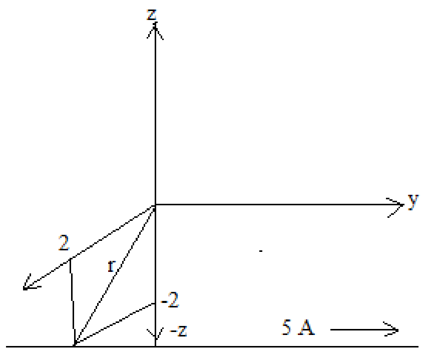


H due to Hexagon is six times the field intensity due to each conductor.

$$H = 6H_1 = 6 \text{ X } \frac{I}{4\pi a} = \frac{1.5\, I}{\pi a} \text{ A/m}$$

5. A current filament of 5A is the a_y direction is parallel to the y-axis at x=2 m, z=-2m. Find H at the origin.

Sol:



The magnetic field due to a straight current filament $H = \frac{I}{2\pi r} a_\emptyset$

$$r = 2\sqrt{2}$$

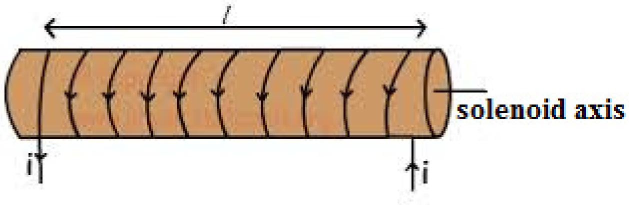
$a_\emptyset = \frac{a_x + a_z}{\sqrt{2}}$ (right hand rule)

$$H = \frac{50}{2\pi 2\sqrt{2}} \left(\frac{a_x + a_z}{\sqrt{2}}\right)$$

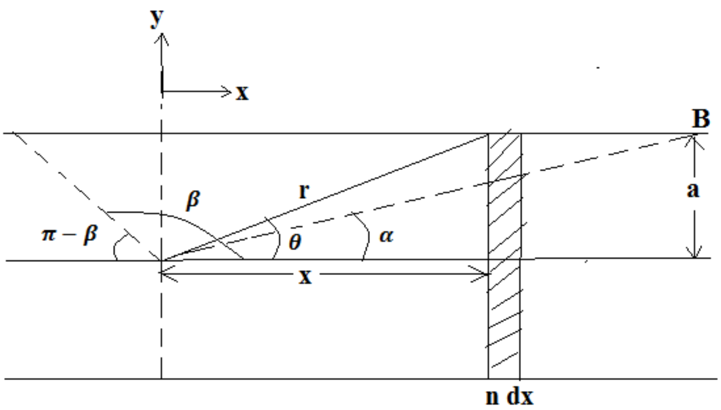
$$= 0.281 \left(\frac{a_x + a_z}{\sqrt{2}}\right) \text{ A/m}$$

3.9. Solenoid:

A solenoid is a cylindrically shaped coil consisting of a large number of closely spaced turns of insulated wire wound usually on a non-magnetic frame.



3.10. H inside a solenoid: (Finite & infinite)



A solenoid has N no of turns on an iron rod. The turns can be assumed to be circular current loops. Considering a small section of length dx at a distance 'x' m from the origin . The no of turns in this section are ndx.

Let dH be the magnetic field intensity due to this section.

We know that $dH = \frac{Ia^2}{2r^3} ndx$ (1)

$\sin\theta = \frac{a}{r}$

$r = \frac{a}{\sin\theta} = a\,\mathrm{cosec}\theta$ (2)

$\cot\theta = \frac{x}{a}$

$x = a\cot\theta$

$dx = -a\,\mathrm{cosec}^2\theta\, d\theta$ (3)

Sub (2) and (3) in (1),

$dH = -\frac{Ia^2}{2a^3\mathrm{cosec}^3\theta}\, n \, X \, a\,\mathrm{cosec}^2\theta\, d\theta$

$= \frac{-In}{2} \frac{1}{cosec\theta}$

$= \frac{-In}{2} \sin\theta d\theta$

Total magnetic field intensity can be obtained by varying θ from β to α.

$H = \int_{\beta}^{\alpha} \frac{-In}{2} \sin\theta d\theta$

$= \frac{-In}{2} \int_{\beta}^{\alpha} \sin\theta d\theta$

$= \frac{In}{2} [\cos\alpha - \cos\beta]$

Case 1: Let 'P' be the midpoint, $\beta = \pi - \alpha$

$H = \frac{In}{2} [\cos\alpha - \cos(\pi - \alpha)]$

$= nI \cos\alpha$

$= \frac{NI}{L} \cos\alpha$

$H = \frac{NI}{L} \cos\alpha$

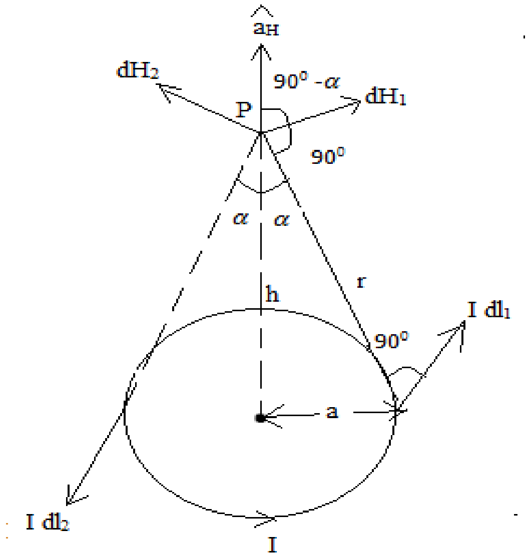
Case 2: For infinitely long solenoid, $\alpha \to 0, \beta = x$

$H = \frac{In}{2} [\cos 0 - \cos \pi]$

$= nI$

$H = \frac{NI}{L}$ AT/m

3.11. H due to a circular current loop:



We have to find H at point 'P' which is at a distance 'h' m from the center of the current loop. The circular loop can be divided into no of current elements. dH_1 and dH_2 are field intensities due to the elements Idl_1 and Idl_2 respectively. Similarly several vectors can be drawn due to several current elements. When these vectors are resolved, radial components get cancelled and normal components get added. There the direction of resultant magnetic field intensity is normal to the plane of the current loop. The same can be obtained using thumb rule (or) cork-screw rule to the current loop.

Normal component due to dH_1 is $dH_1 \cos(90 - \alpha)$ i.e. $dH_1 \sin\alpha$.

Normal component due to dH_2 is $dH_2 \cos(90 - \alpha)$ i.e. $dH_2 \sin\alpha$.

Therefore, sum of normal components would be resultant H_n.

$$H_n = dH_1 \sin\alpha + dH_2 \sin\alpha + \cdots \quad \ldots + dH_n \sin\alpha$$

$$H_n = \int dH_1 \sin\alpha$$

$$H_n = \int \frac{\overrightarrow{Idl} \, X \, \vec{r}}{4\pi|r|^3} \sin\alpha$$

$$= \int \frac{|Idl| \, |r| \, \sin\theta}{4\pi|r|^3} \sin\alpha$$

$$= \int \frac{|Idl| \, |r| \, (1)}{4\pi|r|^3} \sin\alpha$$

$$= \int \frac{I \, dl}{4\pi|r|^2} \left(\frac{a}{r}\right)$$

$$= \frac{I \, a}{4\pi|r|^3} \, X \, 2\pi a$$

$$= \frac{I \, a^2}{2|r|^3}$$

$$|H| = \frac{I\,a^2}{2|r|^3}\ A/m$$

$$\vec{H} = |H|\widehat{a_n}$$

$$H = \frac{I\,a^2}{2(a^2+h^2)^{\frac{3}{2}}}\widehat{a_n}\ A/m$$

To obtain magnetic field intensity at the center of the current loop. At the center, h=0.

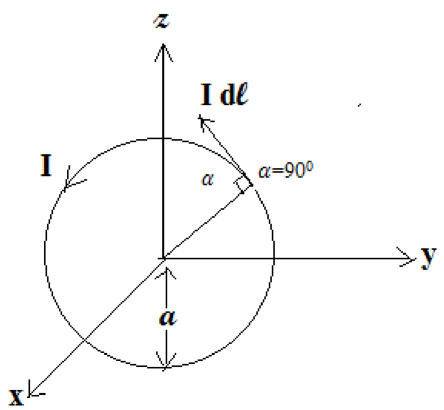
$$H = \frac{I\,a^2}{2a^3} == \frac{I}{2a}\ \widehat{a_n}\ \ A/m$$

If there are N no of turns,

$$\vec{H} = \frac{I\,N\,a^2}{2(a^2+h^2)^{\frac{3}{2}}}\widehat{a_n}\ \ AT/m$$

Q) Find an expression for H (magnetic field intensity) at the center of a circular wire carrying a current I in the anticlockwise direction. The radius of the circle is 'a' and the wire is in x-y plane.

Sol: The direction of the current element I.dl at any point P on the circular wire is given by the tangent at P directed towards the center is along the radius.



$$|dH| = \frac{I\,dl}{4\pi a^2}\sin\theta$$

$$= \frac{I\,dl}{4\pi a^2}$$

$$\overrightarrow{dH} = \frac{\overrightarrow{I\,dl}\ X\ \vec{r}}{4\pi|r|^3}$$

$$\overrightarrow{dH} = \frac{I\,dl\ a\ \sin\theta}{4\pi a^3}\vec{k}$$

$$\overrightarrow{dH} = \frac{I\,dl}{4\pi a^2}\vec{k}$$

$\vec{H} = \frac{\int I\, dl}{4\pi a^2}\vec{k}$

$= \frac{I\,\vec{k}\,(2\pi a)}{\pi a^2}$

$\vec{H} = \frac{I}{2a}\vec{k}\ \text{A/m}$

3.12. Maxwell's second equation:

From Biot-Savart's law, we know that

$\vec{B} = \frac{\mu}{4\pi|r|^3}\left(\overrightarrow{I\,l}\,X\,\vec{r}\right)$

Taking divergence on both sides,

$\text{Div B} = \text{Div}\left(\frac{\mu}{4\pi|r|^3}\right)\left(\overrightarrow{I\,l}\,X\,\vec{r}\right)$

$= \frac{\mu}{4\pi|r|^3}\text{Div}\left(\overrightarrow{I\,l}\,X\,\vec{r}\right)$ (1)

We know that $\text{Div}(\vec{u}\,X\,\vec{v}) = \text{v.curl u} - \text{u.curl v}$ (2)

Using (2), we can write (1) as

$\text{Div B} = \frac{\mu}{4\pi r^3}(\text{r.curl I l} - \text{I l.curl r})$

Curl deals with rotation. The current element vector and distance vector have no rotation. Therefore curl $\overrightarrow{I\,dl}$ and curl $\vec{r}$ vanishes.

$\text{Div B} = \frac{\mu}{4\pi|r|^3}[0 - 0]$

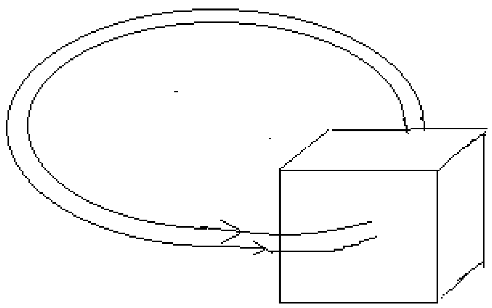
Therefore, Div B= 0.

This equation is known as field form (or) Differential form (or) Vector form of Biot-Savart's law.

This is also known as Maxwell's second equation.

3.13. Alternate proof for Div B=0:

Consider an infinitesimal volume ΔV as shown in fig. From fig, it can be seen that the flux entering and leaving are equal.



Net out flow of flux per unit volume is zero.

$\emptyset = 0$

$\int_s \mathrm{B.ds} = 0$ (1)

From divergence theorem,

$\int_s \mathrm{B.ds} = \int_v \mathrm{Div\,B.dv}$ (2)

From (1) and (2)

$\int_s \mathrm{B.ds} = \int_v \mathrm{Div\,B.dv} = 0$

$\int_v \mathrm{Div\,B.dv} = 0$

Div B=0

Therefore B is a solenoidal field. In electrostatics, positive charge acts as a source and negative charge acts as sink. T he flux lines start from positive charge and terminate on the negative charge. Electric lines of flux are discontinues.

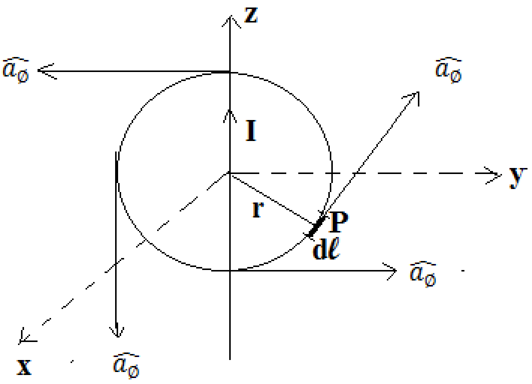
Magnetic lines of flux start at one point and terminate at the same point. These are continuous. This is nothing like a source and sink. Therefore isolated poles do not exist.

3.14. Ampere's law (or) Ampere's circuital law:

The line integral of tangential component of M.F.I vector over a closed path is equal to current enclosed by that path (or) Work done by unit pole around a current carrying conductor is equal to current enclosed. If the conductor has 'N' no of turns,

1) $\oint \mathrm{H.dl} = \mathrm{I_e}$
2) $\mathrm{W.D} = \mathrm{I_e} = \mathrm{NI_e}$
3) $\oint \mathrm{H.dl} = \mathrm{NI_e}$

Proof:



Consider a closed path around a current carrying conductor as shown in fig. The magnetic field at any point on the path is tangent. The point ‘P’ is at a distance ‘r’ from the conductor. Consider dl at point ‘P’ which is at direction $\widehat{a_\emptyset}$ is tangential to the circular path. From the Biot-Savart’s law, the M.F.I along the conductor is given by

$$\overline{H} = \frac{I}{2\pi r}\widehat{a_\emptyset}$$

$$\overline{dl} = d_r\widehat{a_r} + rd\emptyset\widehat{a_\emptyset} + d_z\widehat{a_z}$$

$$\overline{dl} = rd\emptyset\widehat{a_\emptyset}$$

$$\oint H.dl = \oint \frac{I}{2\pi r}\widehat{a_\emptyset}.\,rd\emptyset\,\widehat{a_\emptyset}$$

$$\int_0^{2\pi} \frac{I}{2\pi}\,d\emptyset$$

$$= I$$

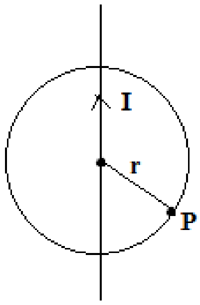
3.15. How to apply ampere,s law:

It is used to determine the value of M.F.I (H) construct an ampere loop such that magnetic field is uniform and direction of magnetic field is tangential to the loop at every point. Then apply Ampere’s law.

Applications:

1) H due to long conductor:

Construct an ampere loop with radius ‘r’ and apply Ampere’s law.



$W.D = I_e$

$\oint H.dl = I_e$

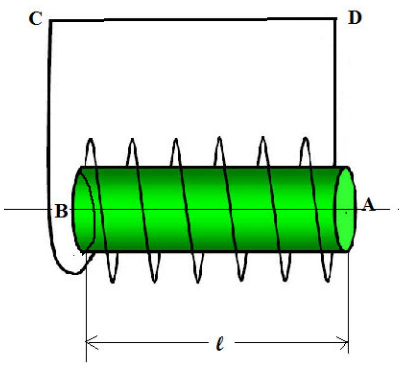
$H.2\pi r = I$

$H = \frac{I}{2\pi r}\widehat{a_\phi}$

Note:

We cannot apply ampere's law to the finite conductor because the magnetic field is not uniform.

2) H due to a long solenoid:



A solenoid has 'N' turns and it carries a current 'I' A. The current direction is shown in fig. The magnetic field is towards left. The magnetic field outside the solenoid is zero.

Consider (or) construct rectangular loop (ABCDA) and apply Ampere's law,

$\oint H.dl = NI$

$\int_{AB} H.dl + \int_{BC} H.dl + \int_{CD} H.dl + \int_{DA} H.dl = NI$

Work done along CD is '0' since it is outside the solenoid.

Work done along DA and BC are '0' because H and dl are perpendicular.

$\int_{AB} H.dl = NI$

$Hl = NI$

$H = \frac{NI}{l}$ AT/m

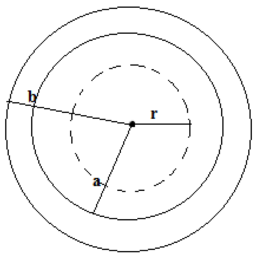
We can't apply ampere's law for finite solenoid because magnetic field is not uniform.

Problems:

1) A long straight tabular conductor with outer diameter 5 cm with wall thickness of 0.5 cm, carries a current of 100 A. Find the magnetic field intensity just inside the wall of the tube and just outside the wall of tube and at a point in the tube half-way between the inner and outer surfaces.

Sol: Given I=100 A

Case 1: Construct ampere loop $r \leq a$



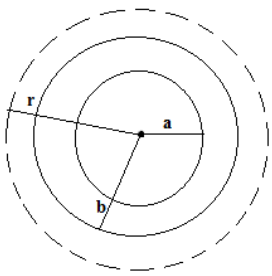
Apply ampere's law.

W.D=I_e

$\oint H.dl = 0$

H=0

Case 2: Construct Ampere's loop $r \geq b$



Apply Ampere's law

W.D=I_e

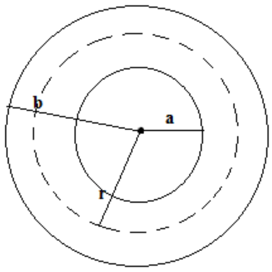
$$\oint H.dl = 100$$

$$H = \frac{100}{2\pi b}$$

$$= \frac{100}{2\pi \, X \, 5 \, X \, 10^{-2}}$$

$$= 636.6 \text{ A/m}$$

Case 3: Construct Ampere's loop $r = 2.25$ cm



Apply Ampere's law

I=100 A

$\pi(b^2 - a^2) = 100$

$J = \frac{I}{A}$

$= \frac{100}{\pi(b^2-a^2)} = \frac{100}{\pi(2.25^2-2^2)} = 14.14$

$I_e = J \, X \, \pi(2.25^2 - 2^2) = 14.14 \, X \, \pi(2.25^2 - 2^2) = 47.19$

Apply ampere's law,

W.D=I_e

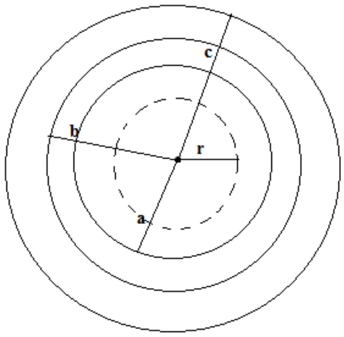
$$\oint H.dl = 0$$

$H. 2\pi r = 47.19$

$H = 333.80 \text{ A/m}$

2) A co-axial transmission line with inner-conductor radius 'a' and outer conductor is annual ring og inner radius 'b', outer radius is 'c' carries a current of '+I' in the center conductor and '-I' in the outer conductor. Find magnetic field due to current distribution.

Sol: **Case 1:** Construct Ampere's loop $r < a$



Apply Ampere's law

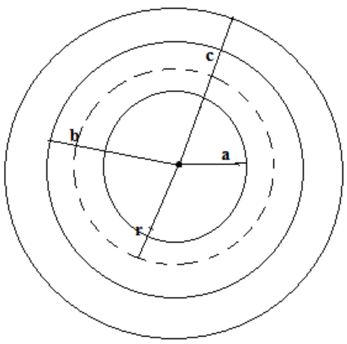
W.D=I_e

$$\oint H.dl = \frac{Ir^2}{a^2}$$

$$H = \frac{Ir^2}{a^2 \, X \, 2\pi r}$$

$$H = \frac{Ir}{2\pi a^2}$$

Case 2: Construct Ampere's loop $a < r < b$



Apply Ampere's law

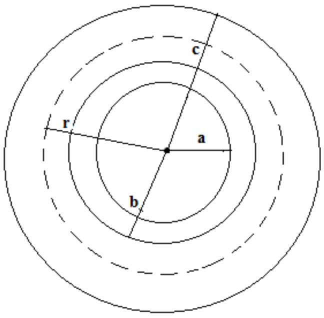
W.D=I_e

$$\oint H.dl = I_e$$

$$H.2\pi r = I$$

$$H = \frac{I}{2\pi r} \ A/m$$

Case 3: Construct ampere's loop with $b < r < c$



Apply Ampere's law

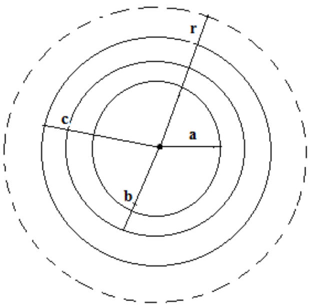
W.D=I_e

$$\oint H.dl = \frac{I(c^2 - r^2)}{(c^2 - b^2)}$$

$$H.2\pi r = \frac{I(c^2 - r^2)}{(c^2 - b^2)}$$

$$H = \frac{I}{2\pi r}\frac{(c^2 - r^2)}{(c^2 - b^2)} \text{ A/m}$$

Case 4: Construct ampere's loop with $r < c$



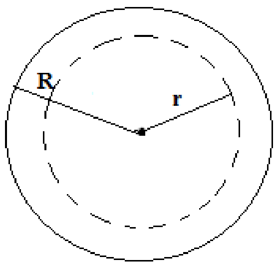
Apply Ampere's law

W.D=I_e

$$\oint H.dl = I - I = 0$$

3) A solid cylindrical conduction of radius 'R' has uniform current density. Determine expression for magnetic field intensity both inside and outside. The conductor float the variation of edge of function of the radius distance from the center of wire.

Sol: **Case 1:** Construct Ampere's loop $r < R$



Apply Ampere’s law

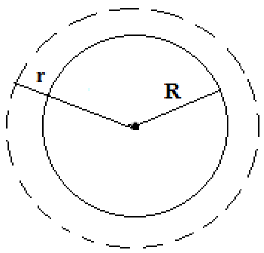
W.D=I_e

$$\oint H.dl = \frac{I}{\pi R^2}.\pi r^2$$

$$\oint H.dl = \frac{Ir^2}{R^2} = I_e$$

$$H = \frac{Ir}{2\pi R^2}\ A/m$$

Case 2: Construct Ampere’s loop $r > R$



Apply Ampere’s law

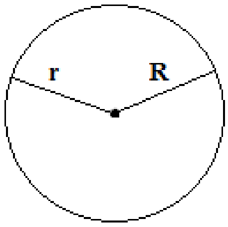
W.D=I_e

$$\oint H.dl = I_e$$

$$H.2\pi r = I$$

$$H = \frac{I}{2\pi r}\ A/m$$

Case 3: Construct Ampere’s loop $R = r$



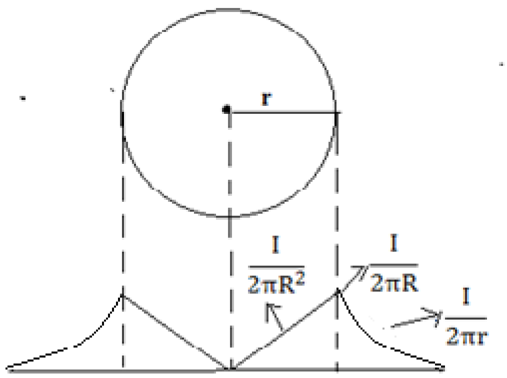
Apply Ampere's law

W.D=I_e

$$\oint H.\,dl = I_e$$

$$H.\,2\pi R = I$$

$$H = \frac{I}{2\pi R}\ A/m$$



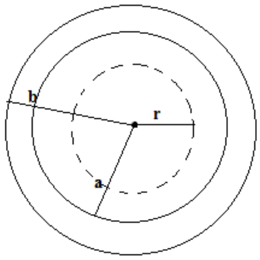
4) A steady current of 100A is flowing in a long straight hollow conductor of inner radius 1 cm and outer radius 2 cm. Assume uniform resistivity. Calculate 'B' , the function of radius 'r' from axes of the conductor.

Sol: Given, a=1 cm

B=2 cm

I=100 cm

Case 1: Construct Ampere's loop $r < a$



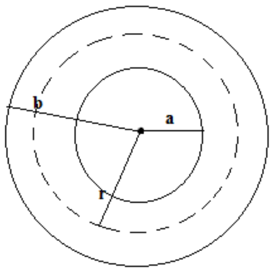
Apply Ampere's law

W.D=I_e

Current inside conductor is '0'

Therefore H=0, B=0.

Case 2: Construct ampere's loop with $b > r > c$



Apply Ampere's law

W.D=I_e

$$\oint \mathrm{H.dl} = \frac{\mathrm{I}(\mathrm{r}^2 - \mathrm{a}^2)}{(\mathrm{b}^2 - \mathrm{a}^2)}$$

$$\mathrm{H}.2\pi\mathrm{r} = \frac{1000(\mathrm{r}^2 - \mathrm{a}^2)}{(\mathrm{b}^2 - \mathrm{a}^2)}$$

$$= \frac{1000(\mathrm{r}^2 - 1 \mathrm{X} 10^{-4})}{(2 \mathrm{X} 10^{-4} - 1 \mathrm{X} 10^{-4})}$$

$$\mathrm{H} = \frac{1}{2\pi\mathrm{r}}(1000 \mathrm{X} 10^4 \mathrm{r}^2 - 1000)$$

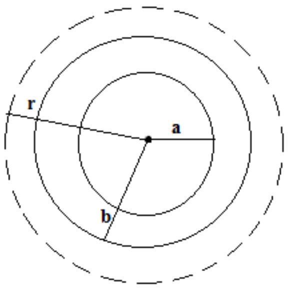
$$\mathrm{H} = 15.91 \mathrm{X} 10^5 \mathrm{r} - \frac{159.55}{\mathrm{r}}$$

$$B = \mu_0 \mathrm{H}$$

$$\mathrm{B} = 4\pi \mathrm{X} 10^{-7}\left(15.9 \mathrm{X} 10^5 \mathrm{r} - \frac{4\pi \mathrm{X} 10^{-7} \mathrm{X} 159.55}{\mathrm{r}}\right)$$

$$\mathrm{B} = 2\mathrm{r} - \frac{2 \mathrm{X} 10^{-4}}{\mathrm{r}} \ \mathrm{Wb/m^2}$$

Case 3: Construct Ampere's loop $r > b$



Apply Ampere's law

$W.D = I_e$

$$\oint H.dl = 1000$$

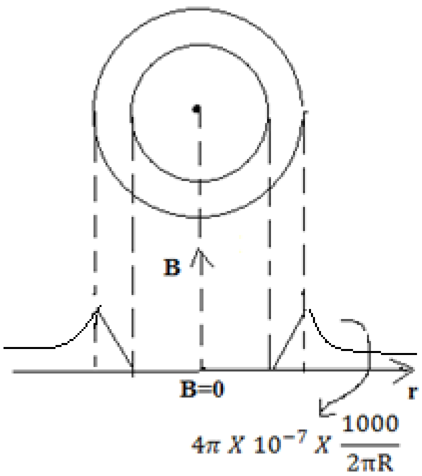
$H.2\pi R = 1000$

$H = \frac{1000}{2\pi R}$

$B = \mu_0 H$

$= 4\pi \, X \, 10^{-7} \, X \, \frac{1000}{2\pi R}$

$B = \frac{2 \, X \, 10^{-4}}{r} \, Wb/m^2$



3.16. Maxwell's third equation:

According to ampere's law, the line integral of magnetic field intensity vector over a closed path is equal to current enclosed by that path.

$$\oint H.dl = I \qquad (1)$$

We know that,

$$I = \int_s J.ds \qquad (2)$$

From eq (1) and (2),

$$\oint H.dl = \int_s J.ds \qquad (3)$$

This is known as integral form of Ampere's law.

From the Stoke's theorem,

We know that

$$\oint H.dl = \int_s \nabla \times H\, ds \qquad (4)$$

From (3) and (4),

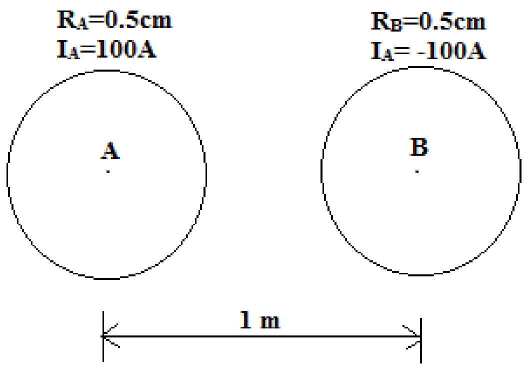
$$\nabla \times H = J \qquad (5)$$

Where 'J' is known as point form (or) vector form (or) differential form of Ampere's law. This is also known as Maxwell's 3rd equation.

Problems:

1) A transmission circuit consists of two parallel conductors A & B of 1 cm diameter each and spaced 1 m apart. The conductors are carrying currents of 100A and -100 A. Determine magnetic field at the surface of each conductor and in space exactly mid way.

Sol: **a) H at surface of conductor:**



Let H_1 and H_2 be magnetic fields at the surface of conductor 'A' due to the current in conductor 'B'.

$$H = H_1 + H_2$$

$$\oint H_1 . dl = I$$

$$H_1 . 2\pi R_A = 100$$

$$H_1 = \frac{100}{2\pi R_A}$$

Similarly,

$$\oint H_2 . dl = 100$$

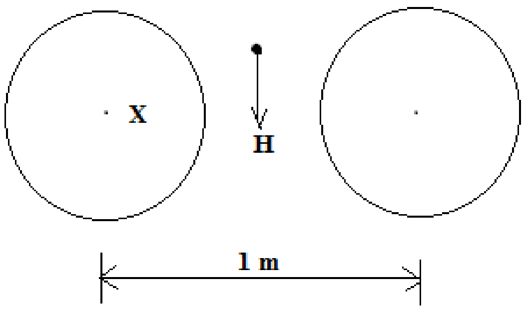
$$H_2 . 2\pi(1 - R_A) = 100$$

$$H_2 = \frac{100}{2\pi(1-R_A)}$$

$$H = H_1 + H_2 = \frac{100}{2\pi R_A}\widehat{a_\emptyset} + \frac{100}{2\pi(1-R_A)}\widehat{a_\emptyset}$$

$$= 3199.1\widehat{a_\emptyset} \text{ A/m}$$

b) H at the center of two conductors:



$$H = H_1 + H_2$$

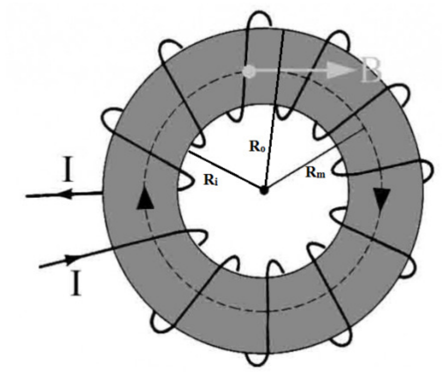
$= \frac{I}{2\pi x}\widehat{a_\emptyset} + \frac{I}{2\pi(d-x)}\widehat{a_\emptyset}$

$= \frac{100}{2\pi \text{ X } 0.5}\widehat{a_\emptyset} + \frac{100}{2\pi(0.5)}\widehat{a_\emptyset}$

$= 63.56\widehat{a_\emptyset} \text{ A/m}$

2) Using Ampere's work law,find the magnetic field intensity in the case of circular solenoid.

Sol:



Mean radius $R_m = \frac{R_i+R_0}{2}$

Construct ampere loop and apply ampere's law of radius 'r'.

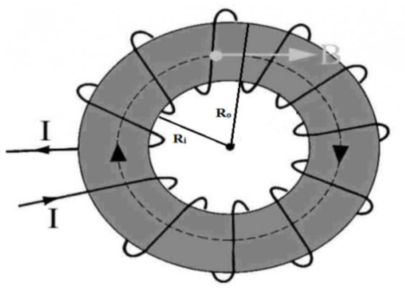
W.D=N I_e
$\oint H.dl = NI$

$H.2\pi R_m = NI$

$H = \frac{NI}{2\pi R_m}$

3) A wood ring of space section of internal diameter 40 cm and external diameter 60 cm carries a toroidal winding of 500 turns uniformly distributed and having current of 1 A. Calculate magnetic field of inner,outer cylindrical surface.

Sol:



outer surface:

W.D=NI

$\oint H.dl = NI$

$H.2\pi R_0 = NI$

$H = \frac{NI}{2\pi R_0}$

$= \frac{500 \text{ X } 1}{2\pi \text{ X } 30 \text{ X } 10^{-2}}$

$= 265$ A/m

Inner surface:

$H = \frac{NI}{2\pi R_i}$

$= \frac{500 \text{ X } 1}{2\pi \text{ X } 20 \text{ X } 10^{-2}}$

$= 397.8$ A/m